By : Dir. Firoz Ahmad

MATHEMATICS

Mob. : 9470844028
9546359990

AIM POINTS
MATHEMATICS
DIR. FIROZ AHMAD

Ram Rajya More, Siwan

XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XI (PQRS)

TRIGONOMETRIC RATIOS OF MULTIPLE AND SUBMULTIPLE ANGLES

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. (i) $\sin 2A = 2 \sin A \cos A$

   (ii) $\cos 2A = \cos^2 A - \sin^2 A$

   (iii) $\cos 2A = 2 \cos^2 A - 1$ or, $1 + \cos 2A = 2 \cos^2 A$

   (iv) $\cos 2A = 1 - 2 \sin^2 A$ or, $1 - \cos 2A = 2 \sin^2 A$

   (v) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

   (vi) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

   (vii) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

2. (i) $\sin 3A = 3 \sin A - 4 \sin^3 A$

   (ii) $\cos 3A = 4 \cos^2 A - 3\cos A$

   (iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

3. (i) $\sin\dfrac{A}{2} = \sqrt{\dfrac{1-\cos A}{2}}$

   (ii) $\cos\dfrac{A}{2} = \sqrt{\dfrac{1+\cos A}{2}}$

   (iii) $\tan\dfrac{A}{2} = \sqrt{\dfrac{1-\cos A}{1+\cos A}}$

4. (i) $\cos A \cos 2A \cos^2 A \cos 2^3 A \ldots \cos 2^{n-1} A = \dfrac{\sin 2^{n-1} A}{2^n \sin A}$

   (ii) $\sin\theta \sin(60° - \theta) \sin(60° + \theta) = \dfrac{1}{4}\sin 3\theta$

   (iii) $\cos\theta \cos(60° - \theta) \cos(60° + \theta) = \dfrac{1}{4}\cos 3\theta$

5. (i) $\sin 18° = \dfrac{\sqrt{5}-1}{4}$

   (ii) $\cos 36° = \dfrac{\sqrt{5}+1}{4}$

   (iii) $\cos 18° = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

(iv) $\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$

EXERCISE-1

1. Prove that :

   (i) $\frac{\cos 2\theta}{1+\sin 2\theta} = \tan\left(\frac{\pi}{4}-\theta\right)$

   (ii) $\frac{\cos\theta}{1+\sin 2\theta} = \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

2. Show that : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

3. Prove that :

   (i) $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$

   (ii) $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=\frac{3}{2}$

4. Prove that :

   (i) $\cos^2 A+\cos^2\left(A+\frac{2\pi}{3}\right)+\cos^2\left(A-\frac{2\pi}{3}\right)=\frac{3}{2}$

   (ii) $\cos^2 A+\cos^2\left(A+\frac{\pi}{3}\right)+\cos^2\left(A-\frac{\pi}{3}\right)=\frac{3}{2}$

5. Prove that : $\cos 4x = 1 - 8\sin^2 x\cos^2 x$.

6. Prove that :

   (i) $\frac{5\sin x-2\sin x+\sin x}{\cos 5x-\cos x}=\tan x$

   (ii) $2\sin x + 2\sin 4x + \sin 6x = 4\cos^2 x\sin 4x$

   (iii) $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

7. Prove that : $\cos A\cos 2A\cos 2^3 A\cos 2^3 A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

8. If $\tan^2\theta = 2\tan^2\phi + 1$, prove that $\cos 2\theta + \sin^2\phi = 0$.

9. If $\tan\frac{\theta}{2}=\sqrt{\frac{1-e}{1+e}}\tan\frac{\phi}{2}$, prove that $\cos\phi=\frac{\cos\theta-e}{1-e\cos\theta}$

10. Find the values of :

    (i) $\cos\frac{\pi}{8}$ (ii) $\sin\frac{\pi}{8}$ (iii) $\tan\frac{\pi}{8}$ (iv) $\sin\frac{\pi}{24}$

(v) $\cos\frac{\pi}{24}$ (vi) $\tan\frac{\pi}{24}$

11. If $0 \le x \le 2\pi$, find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, when :

(i) $\tan x = -\frac{4}{3}$, x lies in quadrant II

(ii) $\cos x = -\frac{1}{3}$, x lies in quadrant III

(iii) $\sin x = -\frac{1}{2}$, x lies in quadrant IV.

12. If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

13. If $\tan\frac{\theta}{2} = \sqrt{\frac{a-b}{a+b}} = \tan\frac{\phi}{2}$, prove that $\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$

14. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha\cos\beta}$, prove that $\tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$

15. If α and β are distinct roots of $a\cos\theta + b\sin\theta = c$, prove that $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$.

16. $(\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cos A = 0$

17. $\cos^2\left(\frac{\pi}{4} - \theta\right) - \sin^2\left(\frac{\pi}{4} - \theta\right) = 2\sin\theta$

19. Prove that : $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2\sec 2\theta$

20. (i) If $\cos x = -\frac{3}{5}$ and x lies in the IIIrd quadrant, find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$, $\sin 2x$.

(ii) If $\cos x = -\frac{3}{5}$ and x lies in IInd quadrant, find the values of $\sin 2x$ and $\sin\frac{x}{2}$.

21. If $\tan A = \frac{1}{7}$ and $\tan B = \frac{1}{3}$, show that $\cos 2A = \sin 4B$.

22. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, prove that :

(i) $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$

(ii) $\cos(\alpha+\beta) = \frac{a^2+b^2-2}{2}$

23. If $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{5}{13}$, prove that $\cos\frac{\alpha-\beta}{2} = \frac{8}{\sqrt{65}}$

24. Prove that :
    (i) $\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$
    (ii) $\cot A + \cot(60° - A) - \cot(60° - A) = 3\cot 3A$

25. Prove that $\frac{\tan 3x}{\tan x}$ never lies between $\frac{1}{3}$ and 3.

26. Prove that : $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 3\theta$

27. $\tan A + \tan(60° + A) - \tan(60° - A) = 3\tan 3A$

28. Prove that $|\cos\theta \cos(60 - \theta) \sin(60 + \theta)| \le \frac{1}{4}$ for all values of θ.

29. Prove that : $\sin 18° = \frac{\sqrt{5}-1}{4}$.

30. Prove that : $\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4}$

31. Prove that : $\sin 12° \sin 48° \sin 54° = \frac{1}{8}$

32. Prove that : $4\sin 27° = \sqrt{(5+\sqrt{5})} - \sqrt{(3-\sqrt{5})}$

EXERCISE-2

Answer each of the following questions in one word or one sentence of as per exact requirement of the questions :

1. If $\cos 4x = 1 + k\sin^2 x \cos^2 x$, then write the value of k.

2. If $\tan\frac{x}{2} = \frac{m}{n}$, then write the value of $m\sin x + n\cos x$.

3. If $\frac{\pi < \frac{3\pi}{2}}{2} < \theta$, then $\frac{\pi}{2} < \theta < \pi$, then write the value of $\sqrt{2\sqrt{2+2\cos 2\theta}}$ in the simplest form.

4. If $\pi < \theta < \frac{3\pi}{2}$, then write the value of $\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$

5. If $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, then write the value of $\sqrt{1-\sin 2\theta}$

EXERCISE-3

Mark the correct alternative in each of the following

1. If $\cos 2x + 2\cos x = 1$ then, $(2 - \cos^2 x)\sin^2 x$ is equal to

   (a) 1 (b) -1 (c) $-\sqrt{5}$ (d) $\sqrt{5}$

2. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, and $\cos 3\theta = \lambda\left(a^3 + \frac{1}{a^3}\right)$, then $\lambda =$

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) none of these

3. If $2\tan\alpha = 3\tan\beta$, then $\tan(\alpha - \beta) =$

   (a) $\frac{\sin 2\beta}{5 - \cos 2\beta}$ (b) $\frac{\cos 2\beta}{5 - \cos 2\beta}$ (c) $\frac{\sin 2\beta}{5 + \cos 2\beta}$ (d) none of these

4. If $\tan\alpha = \frac{1 - \cos\beta}{\sin\beta}$, then

   (a) $\tan 3\alpha = \tan\alpha$ (b) $\tan 2\alpha = \tan\beta$
   (c) $\tan 2\beta = \tan\alpha$ (d) none of these

5. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$, then $\tan\frac{\alpha - \beta}{2} =$

   (a) $-\frac{a}{b}$ (b) $-\frac{b}{a}$ (c) $\sqrt{a^2 + b^2}$ (d) none of these

6. $\tan\theta \sin\left(\frac{\pi}{2} + \theta\right)\cos\left(\frac{\pi}{2} - \theta\right) =$

   (a) 1 (b) -1 (c) $\frac{1}{2}\sin 2\theta$ (d) none of these

7. The value of $2\cos\theta - \cos 3\theta - \cos 5\theta - 16\cos^3\theta \sin^2\theta$ is

   (a) 2 (b) 1 (c) 0 (d) -1

8. If $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = \lambda \sec 2\theta$, then $\lambda =$

   (a) 3 (b) 4 (c) 1 (d) 2

9. The value of $\cos^2\left(\frac{\pi}{6} + \theta\right) - \sin^2\left(\frac{\pi}{6} - \theta\right)$ is

   (a) $\frac{1}{2}\cos 2\theta$ (b) 0 (c) $-\frac{1}{2}\cos 2\theta$ (d) $\frac{1}{2}$

10. The value of $2\sin^2 B + 4\cos(A + B)\sin A \sin B + \cos 2(A + B)$ is

    (a) 0 (b) cos 3A (c) cos 2A (d) none of these

11. If $\tan\frac{\theta}{2}=\sqrt{\frac{1-e}{1+e}}\tan\frac{\alpha}{2}$, then $\cos\alpha =$

(a) $1 - e\cos(\cos\theta + e)$ (b) $\frac{1+e\cos\theta}{\cos\theta-e}$ (c) $\frac{1-e\cos\theta}{\cos\theta-e}$ (d) $\frac{\cos\theta-e}{1-e\cos\theta}$

12. The falue of $\cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta$ is

(a) $\cos 2\theta$ (b) $\sin 2\theta$ (c) $\cos 4\theta$ (d) none of these

13. The value of $\cos(36° - A)\cos(30° + A) + \cos(54° - A)\cos(54° + A)$ is

(a) $\cos 2A$ (b) $\sin 2A$ (c) $\cos A$ (d) 0

14. The value of $\tan\theta + \tan(60° + \theta) + \tan(120° + \theta)$ is

(a) $3\tan 3\theta$ (b) $\tan 3\theta$ (c) $3\cot 3\theta$ (d) $\cot 3\theta$